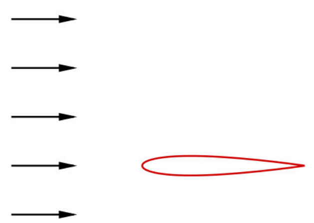
2D Euler Equations : (inviscid)

$P = P_\infty \quad \rho = \rho_\infty \quad u = M_\infty c_\infty \quad v = 0.$



$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0}$$

Mass Conservation

$$\boxed{\begin{aligned} &\frac{\partial \rho u}{\partial t} + \frac{\partial P + \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = 0 \\ &\frac{\partial \rho v}{\partial t} + \frac{\partial \rho u v}{\partial x} + \frac{\partial P + \rho v^2}{\partial y} = 0 \end{aligned}}$$

Momentum Conservation

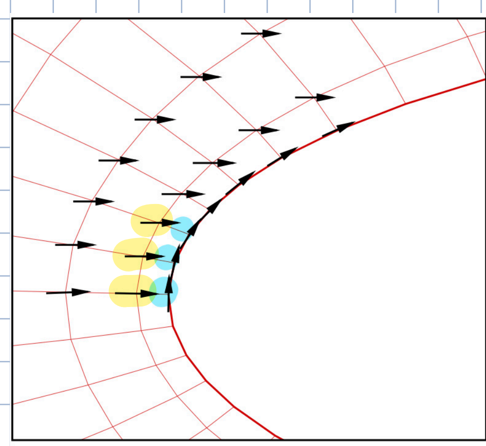
$$\boxed{\frac{\partial E}{\partial t} + \frac{\partial (P+E)u}{\partial x} + \frac{\partial (P+E)v}{\partial y} = 0}$$

Energy Conservation

- Uniform flow initially has zero gradients everywhere.
- Boundary conditions determine what flow evolves from eqns.

↳ e.g for inviscid flow, solid surface boundary condition is tangential flow

For first time step, there will be a large gradient between the uniform flow & surface.



↳ ∴ big error

↳ gradient will propogate out into flow and we must march to get correct sln.

Steps to Solve Euler:

1. Start with guessed sln at $t = 0$ (normally freestream values).
2. Enforce boundary conditions (e.g tangential flow)
3. Evaluate error in spatial gradients at every point: $ERROR = \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y}$
4. If $ERROR \to 0$, finish, otherwise:
5. Evaluate update to conserved variables (ρ in continuity equation):

$$\rho(t + \Delta t) = \rho(t) - \Delta t \times ERROR$$

6. $t = t + \Delta t$, then go to 2 again.

$\frac{\partial \rho}{\partial t} = 0$ for steady

→ This scheme is applied to each of the 4 eqns at every point in mesh.

Numerical Solution Procedures for 'Navier-Stokes Like' Equation:

- Nav. Stokes: 5 non-linear, coupled, parabolic PDE's plus eqn of state

→ when examining numerical methods we use simpler eqns that exhibit same behaviour as NS:

↳ e.g 1D non-linear Burgers'

$$\frac{\partial u}{\partial t} + \frac{\partial \frac{1}{2} u^2}{\partial x} = \alpha \frac{\partial^2 u}{dx^2}$$

u is any scalar

→ Same essential features: temporal derivative, non-linear spatial deriv. and a diffusive term

→ There are also analytical slns. available for some forms of eqn. ∴ we can compare numerical slns.

We can write Burgers' in non-conservative form:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \alpha \frac{\partial^2 u}{\partial x^2}$$

→ can set α (viscocity coeff.) to zero and set wavespeed to constant 'c'.

↳ for inviscid

$\hookrightarrow \quad \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$ (linear wave equation)

if at $t=0$, $u(x,0) = f(x)$, then $u(x,t) = f(x-ct)$

$\hookrightarrow$ wave at constant speed c

We will use this as test case for our numerical methods:

1. Guess $u(x,0)$
2. Enforce BC's
3. Let solution 'evolve' (time-march) over small Δt to new sln. $u(x, t+\Delta t)$
4. Evaluate $\frac{\partial u}{\partial t}$ ($= -c\frac{\partial u}{\partial x}$ here)
5. If $\frac{\partial u}{\partial t}$ small we have steady state sln.
6. If not, set $t = t + \Delta t$ and go to 2.

(sometimes seemingly steady problems have unsteady solution e.g 0° aerofoil can have oscillating shockwaves)